Planes, Products, & Distances Cont.

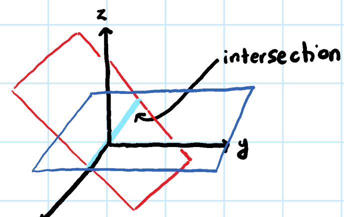
Monday, May 15, 2023 9:32 AM

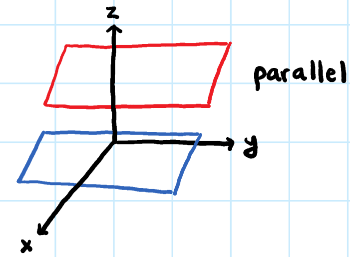
second application:

using $\times$ to determine intersection of planes:

2 planes $\pi = \{ax + by + cz = d\}$
$\pi' = \{\alpha x + \beta y + \gamma z = \delta\}$ different planes

how can they intersect? parallel / don't intersect; line





how to decide & find $\pi \cap \pi'$? intersection

- if $\langle a, b, c\rangle$ parallel to $\langle \alpha, \beta, \gamma\rangle$, then π parallel to π' if & only if $\langle a, b, c\rangle \times \langle \alpha, \beta, \gamma\rangle = 0$

* if orthogonal vectors $\parallel$, then planes $\parallel$ *

- else, $\pi \cap \pi'$ is line with direction:

$$\langle a, b, c\rangle \times \langle \alpha, \beta, \gamma\rangle$$

ex) find intersection of ...

$\pi = \{3x - 4y + z = 17\}$ &

$\pi' = \{$plane containing $\langle 1,1,0\rangle$ & $\langle 0,0,1\rangle$ thru $P = (3,0,-1)\}$ ← description 4

solution: find $\perp$ directions & "$\times$"

for π: $\vec{n} = \langle 3, -4, 1\rangle$

for π': $\vec{n}' = \vec{u} \times \vec{v} = \langle 1,1,0\rangle \times \langle 0,0,1\rangle = \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \langle 1, -1, 0\rangle$

for intersections: $\vec{n} \times \vec{n}' = \langle 3, -4, 1\rangle \times \langle 1, -1, 0\rangle = \begin{vmatrix} i & j & k \\ 3 & -4 & 1 \\ 1 & -1 & 0 \end{vmatrix} = \langle 1, 1, 1\rangle$

(direction of line $\pi \cap \pi'$)